Enlarging the Terminal Region of NMPC with Parameter-Dependent Terminal Control Law

Shuyou Yu, Hong Chen, Christoph Böhm, and Frank Allgöwer

Abstract. Nominal stability of a quasi-infinite horizon nonlinear model predictive control (QIH-NMPC) scheme is obtained by an appropriate choice of the terminal region and the terminal penalty term. This paper presents a new method to enlarge the terminal region, and therefore the domain of attraction of the QIH-NMPC scheme. The proposed method applies a parameter-dependent terminal controller. The problem of maximizing the terminal region is formulated as a convex optimization problem based on linear matrix inequalities. Compared to existing methods using a linear time-invariant terminal controller, the presented approach may enlarge the terminal region significantly. This is confirmed via simulations of an example system.

Keywords: Nonlinear Model predictive control; Terminal invariant sets; Linear differential inclusion; Linear matrix inequality.

1 Introduction

Nonlinear model predictive control (NMPC) is a control technique capable of dealing with multivariable constrained control problems. One of the main stability results for NMPC is the quasi-infinite horizon approach [1, 2]. A remainding issue for QIH-NMPC is how to enlarge the terminal region since the size of the terminal region affects directly the size of the domain of attraction for the nonlinear optimization problem. Many efforts have been made to determine the terminal penalty term and the associated terminal controller such that the terminal region is enlarged.

Shuyou Yu, Christoph Böhm, and Frank Allgöwer
Institute for Systems Theory and Automatic Control, University of Stuttgart, Germany
e-mail: {shuyou,cboehm,allgower}@ist.uni-stuttgart.de

Shuyou Yu and Hong Chen
Department of Control Science and Engineering, Jilin University, China
e-mail: chenh@jlu.edu.cn

L. Magni et al. (Eds.): Nonlinear Model Predictive Control, LNCIS 384, pp. 69–78.
springerlink.com

For nonlinear systems, using either a local polytopic linear differential inclusions (LDI) representation [3] or a local norm-bounded LDI representation [4], the terminal region is obtained by solving an linear matrix inequality (LMI) optimization problem. In [5], a local LDI representation is used as well, and a polytopic terminal region and an associated terminal penalty are computed. Using support vector machine learning [6], freedom in the choice of the terminal region and terminal penalty needed for asymptotic stability is exploited in [6].

Here, we generalize the scheme in [7]. A more general polytopic LDI description is used to capture the nonlinear dynamics and the condition of twice continuous differentiability of the nonlinear system is relaxed to continuous differentiability. The approach results in a parameter-dependent terminal control law. Compared with the use of time-invariant linear state feedback laws, the proposed approach provides more freedom in the choice of the terminal region and terminal cost needed for asymptotic stability. Thus a larger terminal region is obtained.

The remainder of this paper is organized as follows. Section 2 briefly introduces the QIH-NMPC scheme. The condition to calculate terminal region of QIH-NMPC based on linear differential inclusions and the optimization algorithm to maximize the terminal region are proposed in Section 3 and 4. The efficacy of the algorithm is illustrated by a numerical example in Section 5.

2 Preliminaries

Consider the smooth nonlinear control system

$$\dot{x}(t) = f(x(t), u(t)), \quad x(t_0) = x_0, \quad t \geq t_0 \tag{1a}$$

$$z(t) = g(x(t), u(t)), \tag{1b}$$

subject to

$$z(t) \in Z \subset \mathbb{R}^p, \quad \forall t \geq t_0, \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ are the state and input vector, and $z(t)$ is the output vector. Denote X and U as the projection of the output vector space Z to the state vector space and the input vector space, respectively.

Fundamental assumptions of (1) are as follows:

A0) The nonlinear fuctions f and g are continuously differentiable, and satisfy $f(0,0) = 0$ and $g(0,0) = 0$. The equilibrium is a hyperbolic fixed point.

A1) System (1) has a unique solution for any initial condition $x_0 \in X$ and any piecewise right-continuous input function $u(\cdot) : [0, T_p] \longrightarrow U$;

A2) $U \subset \mathbb{R}^m$ and $X \subseteq \mathbb{R}^n$ are compact and the point $(0,0)$ is contained in the interior of $X \times U$.

For the actual state $x(t)$, the optimization problem in the QIH-NMPC is formulated as follows [2, 8]:

$$\min_{\bar{u}(\cdot)} J\big(x(t), \bar{u}(\cdot)\big) \tag{3}$$

subject to

$$\dot{\bar{x}} = f(\bar{x}, \bar{u}), \quad \bar{x}(t; x(t)) = x(t), \tag{4a}$$

$$\bar{z}(\tau) \in Z, \quad \tau \in [t, t+T_p], \tag{4b}$$

$$\bar{x}(t+T_p; \bar{x}(t)) \in \Omega(\alpha), \tag{4c}$$

where $J(x(t), \bar{u}(\tau, x(t))) = V(\bar{x}(t+T_p); x(t)) + \int_t^{t+T_p} F\big(\bar{x}(\tau; x(t)), \bar{u}(\tau)\big) d\tau$, T_p is the prediction horizon, $\bar{x}(\cdot; x(t))$ denotes the state trajectory starting from the current state $x(t)$ under the control $\bar{u}(t)$. The pair $(\bar{x}, \bar{u})$ denotes the optimal solution to the open-loop optimal control problem (3). $F(\cdot,\cdot)$ is the stage cost satisfying the following condition:

A3) $F(x,u) : \mathbb{R}^n \times U \to \mathbb{R}$ is continuous and satisfies $F(0,0) = 0$ and $F(x,u) > 0$, $\forall (x,u) \in \mathbb{R}^n \times U \setminus \{0,0\}$.

In (4), the set $\Omega(\alpha)$ is a neighborhood of the origin and defined as a level set of a positive definite function $V(\cdot)$ as follows

$$\Omega(\alpha) := \{x \in R^n \mid V(x) \le \alpha\}. \tag{5}$$

Moreover, $\Omega(\alpha)$ and $V(x)$ are said to be the terminal region and the terminal penalty respectively, if there exists a continuous local controller $u = \kappa(x)$ such that the following conditions are satisfied:

B0) $\Omega(\alpha) \subseteq X$,
B1) $g(x, \kappa(x)) \in Z$, for all $x \in \Omega(\alpha)$,
B2) $V(x)$ satisfies inequality

$$\frac{\partial V(x)}{\partial x} f(x, \kappa(x)) + F(x, \kappa(x)) \le 0, \qquad \forall x \in \Omega(\alpha). \tag{6}$$

Clearly, $\Omega(\alpha)$ has the following additional properties [8]:

- The point $0 \in \mathbb{R}^n$ is contained in the interior of $\Omega(\alpha)$ due to the positive definiteness of $V(x)$ and $\alpha > 0$,
- $\Omega(\alpha)$ is closed and connected due to the continuity of V in x.
- Since (6) holds, $\Omega(\alpha)$ is invariant for the nonlinear system (1) controlled by local control $u = \kappa(x)$.

The following stability results can be established:

Lemma 1. *[8] Suppose that*

(a) assumptions A0)-A3) are satisfied,
(b) for the system (1), there exist a locally asymptotically stabilizing controller $u = \kappa(x)$, a continuously differentiable, positive definite function $V(x)$ that satisfies (6) for $\forall x \in \Omega(\alpha)$ and a terminal region $\Omega(\alpha)$ defined by (5),
(c) the open-loop optimal control problem described by (3) is feasible at time $t = 0$.

Then, the closed-loop system is nominally asymptotically stable with the region of attraction D being the set of all states for which the open-loop optimal control problem has a feasible solution.

3 Enlarging the Terminal Region of Quasi-infinite Horizon NMPC

In this section we derive a sufficient condition for the calculation of the terminal region and a linear parameter-dependent terminal control law based on a polytopic differential inclusion description of the nonlinear system (1). The constraints under consideration are

$$-\hat{z}_k \leq z_k(t) \leq \hat{z}_k, \quad k = 1,2,\ldots,p, \quad t \geq t_0, \tag{7}$$

where $z_k(\cdot)$ is the kth element of the outputs, and $\hat{z}_k$ is positive scalar.

We choose the stage cost $F(x,u) = x^T Qx + u^T Ru$ with $0 \leq Q \in R^{n\times n}$ and $0 \leq R \in^{m\times m}$. Suppose that the Jacobian linearization of the system (1) at the origin is stabilizable. Then a quadratic Lyapunov function and a local region round the equilibrium defined by the level set of the Lyapunov function exist [9] which serve as terminal penalty and terminal region, respectively. Therefore, we choose the terminal region $\Omega(\alpha,P) := \{x \in R^n | x^T Px \leq \alpha\}$ which represents an ellipsoid.

3.1 Polytopic Linear Differential Inclusions

Suppose that for each x,u and t there is a matrix $G(x,u,t) \in \Pi$ such that

$$\begin{bmatrix} f(x,u) \\ g(x,u) \end{bmatrix} = G(x,u,t) \begin{bmatrix} x \\ u \end{bmatrix} \tag{8}$$

where $\Pi \subseteq R^{(n+p)\times(n+p)}$. If we can prove that every trajectory of the LDI defined by Π has some property, then we have proved that every trajectory of the nonlinear system (1) has this property. Conditions that guarantee the existence of such a G are $f(0,0) = 0$, $g(0,0) = 0$, and $\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial u} \\ \frac{\partial g}{\partial x} & \frac{\partial f}{\partial u} \end{bmatrix} \in \Pi$ for all x,u,t [10].

The set Π is called a polytopic linear differential inclusion (PLDI) if Π is described by a list of its vertices [10]

$$\Omega = \mathrm{Co}\left\{ \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}, \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}, \ldots, \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix} \right\}, \tag{9}$$

where $\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$, $i = 1,2,\ldots,N$ are vertex matrices of the set Π, and N is the number of vertex matrices. Then the nonlinear system (1) can be represented in the form of a linear parameter-varying dynamic system [11]

$$\dot{x}(t) = \sum_{i=1}^{N} \beta_i(\lambda)(A_i x(t) + B_i u(t)), \tag{10a}$$

$$z(t) = \sum_{i=1}^{N} \beta_i(\lambda)\,(C_i x(t) + D_i u(t)) \tag{10b}$$

where $\lambda \in \mathbb{R}^{n_\lambda}$ is the time-variant parameter vector, and $\beta_i(\lambda)$ are non-negative scalar continuous weighting functions satisfying $\beta_i(\lambda) > 0$ and $\sum_{i=1}^{N} \beta_i(\lambda) = 1$. In the following we denote $\beta(\lambda) = \left[\beta_1(\lambda)\ \beta_2(\lambda)\ \dots\ \beta_N(\lambda)\right]^T$. Suppose that $K_j \in \mathbb{R}^{m\times n}$ is a time-invariant feedback gain of the ith vertex system, the control law for the whole PLDI system can be inferred as a weighted average of controllers designed for all vertices, i.e.,

$$\kappa(\lambda) = \sum_{j=1}^{N} \beta_j(\lambda) K_j. \tag{11}$$

Substituting (11) into (10), we obtain the closed-loop system

$$\dot{x}(t) = A_{cl}\big(\beta(\lambda)\big)x(t), \tag{12a}$$

$$z(t) = C_{cl}\big(\beta(\lambda)\big)x(t), \tag{12b}$$

with $A_{cl}\big(\beta(\lambda)\big) = \sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} \beta_i(\lambda)\beta_j(\lambda)(A_i + B_i K_j)$, $C_{cl}\big(\beta(\lambda)\big) = \sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} \beta_i(\lambda)\beta_j(\lambda)$ $(C_i + D_i K_j)$.

3.2 Terminal Region of NMPC Based on PLDI

Based on the PLDI of nonlinear system (1), the inequality condition (6) can be formulated as a linear matrix inequality (LMI) problem. This is attractive since computationally efficient methods to solve such problems are available [10, 11].

Theorem 1. *For system (12), suppose that there exist a matrix $X > 0$ and matrices Y_j such that*

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \beta_i(\lambda)\beta_j(\lambda) \begin{bmatrix} A_i X + B_i Y_j + (A_i X + B_i Y_j)^T & X & Y_i^T \\ X & -Q^{-1} & 0 \\ Y_j & 0 & -R^{-1} \end{bmatrix} \leq 0, \tag{13}$$

Then, with $\kappa(\lambda) = \sum_{j=1}^{N} \beta_j(\lambda) K_j$ as in (11) and $V(x) := x^T P x$, where $P = X^{-1}$ and $K_j = Y_j X^{-1}$, the inequality (6) is satisfied.

Proof: By substituting $P = X^{-1}$ and $Y_j = K_j X$ in (13) and performing a congruence transformation with the matrix $\{X^{-1}, I, I\}$, we obtain

$$\begin{bmatrix} A_{cl}(\lambda)^T P + P A_{cl}(\lambda) & X & \kappa(\lambda)^T \\ X & -Q^{-1} & 0 \\ \kappa(\lambda) & 0 & -R^{-1} \end{bmatrix} \leq 0,$$

It follows from the Schur complement that the inequalities (13) are equivalent to

$$A_{cl}(\lambda(t))^T P + P A_{cl}(\lambda(t)) + Q + \kappa(\lambda(t))^T R \kappa(\lambda(t)) \leq 0. \tag{14}$$

We choose $V(\xi) = \xi^T P \xi$ as a Lyapunov function candidate. The time derivative of $V(x)$ along the trajectory of (12) is given as follows:

$$\begin{aligned}\frac{dV(x(t))}{dt} &= \dot{x}(t)^T P x(t) + x(t)^T P \dot{x}(t) \\ &= x(t)^T \Big\{ \sum_{i=1}^{N}\sum_{j=1}^{N} h_i(\lambda)h_j(\lambda)\Big((A_i + B_i K_j)^T P + P(A_i + B_i K_j)\Big)\Big\} x(t) \\ &= x(t)^T \Big\{ A_{cl}(\lambda)^T P + P A_{cl}(\lambda)\Big\} x(t) \end{aligned} \tag{15}$$

Using (14), we have $\frac{dV(x(t))}{dt} \le -x(t)^T Q x(t) - x(t)^T \kappa(\lambda)^T R \kappa(\lambda) x(t)$. Thus the inequality (6) holds, and $\kappa(\lambda)$ is the associated terminal control law. □

Now we derive conditions such that the constraints (7) are satisfied by the controller $\kappa(\lambda)$ for any $x \in \Omega(P,\alpha)$.

Theorem 2. *If X and $Y_j, j = 1,2,\ldots,N$ satisfy (13) and furthermore the following matrix inequalities*

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \beta_i(\lambda)\beta_j(\lambda) \begin{bmatrix} \frac{1}{\alpha}\hat{z}_k^2 & e_k^T(C_i X + D_i Y_j) \\ * & X \end{bmatrix} \ge 0, k = 1,2,\ldots,p, \tag{16}$$

hold, where e_k is kth element of the basis vector in the constraint vector space, then for any $x(t) \in \Omega(P,\alpha)$, the parameter-dependent feedback law (11) controls the system (12) satisfying the the constraint (7).

Proof: Using (12b), satisfaction of the constraints (7) requires

$$x(t)^T (C_{cl}(\beta(\lambda))^T e_k e_k^T C_{cl}(\beta(\lambda)) x(t) \le \hat{z}_k^2, \tag{17}$$

due to $x(t) \in \Omega(P,\alpha)$, which holds if

$$\frac{x(t)^T (C_{cl}(\beta(\lambda))^T e_k e_k^T C_{cl}(\beta(\lambda)) x(t)}{\hat{z}_k^2} \le \frac{x(t)^T P x(t)}{\alpha}, \tag{18}$$

For any $x(t) \neq 0$ (note that $x(t) = 0$ leads to $z(t) = 0$ and satisfaction of (7)), inequality (18) holds if

$$\frac{P}{\alpha} - \frac{(C_{cl}(\beta(\lambda))^T e_k e_k^T C_{cl}(\beta(\lambda))}{\hat{z}_k^2} \ge 0. \tag{19}$$

Applying the Schur Complement, the matrix inequality (19) is equivalent to

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \beta_i(\lambda)\beta_j(\lambda) \begin{bmatrix} P & * \\ e_k^T(C_i + D_i K_j) & \frac{1}{\alpha}\hat{z}_k^2 \end{bmatrix} \ge 0, \quad k = 1,2,\cdots,p. \tag{20}$$

Performing a congruence transformation with $\mathrm{diag}(I,X)$ on both sides of (20), we obtain the required inequality (16). □

Following the above discussions, we now state the main results of this paper:

Theorem 3. *Suppose that the PLDI model of the nonlinear system (1) is given by (12). If there exist a positive definite matrix $X \in \mathbb{R}^{n\times n}$, matrices $Y_j \in \mathbb{R}^{m\times n}$, $j = 1,2,\cdots,N$, and a scalar $\alpha > 0$, independent of the unknown parameter vector $\beta(\lambda)$ such that (13) and (16), then the ellipsoid $\Omega(\alpha,P)$ with $P = X^{-1}$ and $V(x) = x^T Px$ serve as a terminal region and a terminal penalty for NMPC, respectively. The associated terminal controller is $\kappa(\lambda) = \sum_{j=1}^{N} \beta_j(\lambda)K_j x(t)$ with $K_j = Y_j X^{-1}$.*

Proof: The inequalities (13) and (16) guarantee that the nonlinear system (1) satisfy inequality (6) and constraints (2), respectively, i.e. B1) and B2).

The positive definite matrix $X \in \mathbb{R}^{n\times n}$, the matrices $Y_j \in \mathbb{R}^{m\times n}$, and the scalar $\alpha > 0$ are independent of the unknown parameter vector $\beta(\lambda)$. Thus $\Omega(\alpha,P)$ is the terminal region and $V(x)$ is the terminal penalty of the nonlinear system, respectively. □

4 Optimization of the Terminal Region

In order to reduce the functional inequalities (13) and (16) to finitely many LMIs, we utilize the following lemma:

Lemma 2. *[12] If there exist matrices $\Gamma_{ii} = \Gamma_{ii}^T$, $\Gamma_{ij} = \Gamma_{ji}^T$, $(i \neq j, i, j = 1,2,\cdots,r)$ such that the matrix $\Lambda_{ij}(1 \leq i,j \leq r)$*

$$\Lambda_{ii} \leq \Gamma_{ii}, \quad i = 1,2,\cdots,r, \tag{21a}$$

$$\Lambda_{ij} + \Lambda_{ji} \leq \Gamma_{ij} + \Gamma_{ij}^T, \quad j < i, \tag{21b}$$

$$[\Gamma_{ij}]_{r\times r} \leq 0, \tag{21c}$$

then the parameter matrix inequalities

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \delta_i(\cdot)\delta_j(\cdot)\Lambda_{ij} \leq 0, \tag{22}$$

are feasible, where $\delta_i(\cdot) \geq 0$, $\sum_{i=1}^{r} \delta_i(\cdot) = 1, \forall t$, and $[\Gamma_{ij}]_{r\times r} = \begin{pmatrix} \Gamma_{11} & \cdots & \Gamma_{1r} \\ \vdots & \ddots & \vdots \\ \Gamma_{r1} & \cdots & \Gamma_{rr} \end{pmatrix}$.

Let $\Omega(\alpha,P)$ denote the ellipsoid centered at the origin defined by P and α. The volume of Ω is proportional to $\det(\alpha X)$, $X = P^{-1}$ [10]. The geometric mean of the eigenvalues [13], leading to minimization of $\det(\alpha X)^{\frac{1}{n}}$, where n is dimension of X, can be used for solving the determinant maximization problem. Define

$$X_0 = \alpha X, \quad Y_{j0} = \alpha Y_j. \tag{23}$$

The inequality constraints (13), (16) can be rewritten as

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \beta_i(\lambda)\beta_j(\lambda)\mathscr{L}_{ij} \leq 0, \tag{24}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\beta_i(\lambda)\beta_j(\lambda)\mathscr{F}_{i,j}\geq 0, \quad k=1,2,\ldots,m, \tag{25}$$

where $\mathscr{L}_{ij}=\begin{pmatrix}\Xi & X_0 & Y_{j0}^T\\ * & -\alpha Q^{-1} & 0\\ * & * & -\alpha R^{-1}\end{pmatrix}$, $\mathscr{F}_{i,j}=\begin{bmatrix}\hat{z}_k^2 & e_k^T(C_iX_0+D_iY_{j0})\\ * & X_0\end{bmatrix}$, $\Xi=X_0A_i^T+Y_{j0}^TB_i^T+A_iX_0+B_iY_{j0}$.

It follows from Lemma 2 that if there exist matrices T_{ij} $(i,j=1,\cdots,N)$ such that

$$\mathscr{L}_{ii}\leq\mathscr{T}_{ii}, \quad i=1,2,\cdots,N, \tag{26a}$$

$$\mathscr{L}_{ij}+\mathscr{L}_{ji}\leq\mathscr{T}_{ij}+\mathscr{T}_{ij}^T, \quad j<i, \tag{26b}$$

$$[\mathscr{T}_{ij}]_{N\times N}\leq 0, \tag{26c}$$

then the inequality (6) is satisfied. Furthermore, if there exist matrices M_{ij} $(i,j=1,2,\cdots,N)$ such that

$$\mathscr{F}_{ii}\geq\mathscr{M}_{ii}, \quad i=1,2,\cdots,N, \tag{27a}$$

$$\mathscr{F}_{ij}+\mathscr{F}_{ji}\geq\mathscr{M}_{ij}+\mathscr{M}_{ij}^T, \quad j<i, \tag{27b}$$

$$[\mathscr{M}_{ij}]_{N\times N}\geq 0, \tag{27c}$$

then the output constraints (7) are satisfied.

Hence, the maximization problem of the ellipsoid Ω can be reformulated as

$$\max_{\alpha,\,X_0,\,Y_{j0}}(\det X_0)^{\frac{1}{n}}, \text{ s.t. } \alpha>0,\ X_0>0,\ (26) \text{ and } (27). \tag{28}$$

Solving the convex optimization problem (28), the optimal solutions $X_0, Y_{j0}, (j=1,\cdots,N)$ and α are determined. The matrices X and Y_j can be found from (23). Then the optimal terminal penalty matrix P, the terminal region Ω, and the terminal feedback law can be determined by Theorem 3. Sometimes solving the optimization problem (28) gives a very large terminal penalty such that the effect of the integral term in the performance index (3) almost disappears. A very strong penalty on the terminal states may have a negative influence on achieving the desired performance which is specified by the finite horizon cost [2]. The trade off between a large terminal region and good control performance can be made by limiting the norm of the matrix P [3]. Since $P=X^{-1}=\alpha(X_0)^{-1}$, it can be achieved by imposing the requirement that α has to be less than or equal to a given constant.

5 A Numerical Example

In this section, the proposed method is applied to a continuous stirred tank reactor(CSTR) [14]. Assuming constant liquid volume, the CSTR for an exothermic, irreversible reaction, A→B, is described by the following dynamic model based on a component balance for reactant A and an energy balance:

$$\dot{C}_A = \frac{q}{V}(C_{Af} - C_A) - k_0 \exp(-\frac{E}{RT})C_A,$$
$$\dot{T} = \frac{q}{V}(T_f - T) - \frac{\triangle H}{\rho C_p} k_0 \exp(-\frac{E}{RT})C_A + \frac{UA}{V\rho C_p}(T_c - T).$$

here C_A is the concentration of the reactor, T is the reactor temperature, and T_c is the temperature of the coolant steam. The parameters are $q = 100$ l/min, $V = 100$ l, $C_{Af} = 1$ mol/l, $T_f = 350$ K, $\rho = 10^3$ g/l, $C_p = 0.239$ J/(g K), $k_0 = 7.2 \times 10^1 0\ \mathrm{min}^{-1}$, $E/R = 8750$ K, $\triangle H = -5 \times 10^4$ J/mol, $UA = 5 \times 10^4$ J/(min K). Under these conditions the steady state is $C_A^{eq} = 0.5$ mol/l, $T_c^{eq} = 300$ K, and $T^{eq} = 350$ K, which is an unstable equilibrium. The temperature of the coolant steam is constrained to $250\ \mathrm{K} \leq T_c \leq 350\ \mathrm{K}$. The concentration of the reactor has to satisfy $0 \leq C_A \leq 1$ mol/l, and the temperature of the reactor is constrained to $300\ \mathrm{K} \leq T \leq 400$ K. The objective is to regulate the concentration C_A and the reactor steam temperature T around the steady state via NMPC by using the temperature of the coolant as an input, while the constraints have to be hold. The dynamics of the CSTR can be expressed by the parameter-dependent weighted linear model of the nonlinear system (10) with $A_1 = \begin{bmatrix} -23.7583 & 0 \\ 4761.2 & -739/239 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1.0155 & 0 \\ 3.2433 & -739/239 \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 0 & 500/239 \end{bmatrix}^T$. The weighting matrices of the stage cost are $Q = \begin{bmatrix} \frac{1}{0.5} & \frac{1}{350} \end{bmatrix}$ and $R = \frac{1}{300}$, respectively.

The volume of the terminal region of the proposed method is compared to previous results which were based on a Lipschitz approach [2]. In order to preserve a dominating effect of the integral part in the cost function, we impose the constraint $\alpha \leq 5$ on the optimization problem (28). The terminal region given by [2] is represented by the dashed ellipsoid, and the terminal region yielded by the PLDI

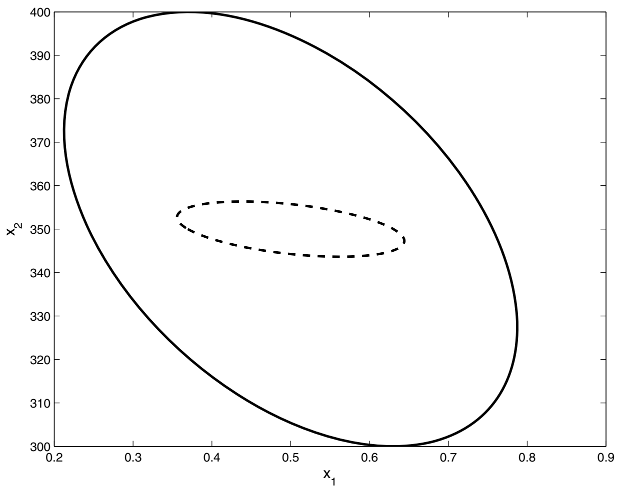

Fig. 1 Comparison of the terminal region

approach with parameter-dependent terminal control law is shown by the solid ellipsoid in Figure 1. The associated terminal penalty is $P = \begin{bmatrix} 15.2100 & 0.0395 \\ 0.0395 & 0.0005 \end{bmatrix}$.

6 Conclusions

In this paper we propose a method to expand the terminal region which replaces the time invariant linear state feedback control law by a parameter-dependent terminal control law. The new algorithm provides an extra degree of freedom to enlarge the terminal set. The problem of maximizing the terminal region is formulated as an LMI based optimization problem. It is shown that, compared to the algorithms with static linear terminal control law, a parameter-dependent terminal control results in a larger terminal region, which is confirmed by a numerical example.

References

1. Mayne, D.Q., Rawlings, J.B., Rao, C.V., Scokaert, P.O.M.: Constrained model predictive control: Stability and optimality. Automatica 36(6), 789–814 (2000)
2. Chen, H., Allgöwer, F.: A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability. Automatica 34(10), 1205–1217 (1998)
3. Chen, W.H., O'Relly, J., Ballance, D.J.: On the terminal region of model predictive control for non-linear systems with input/state constraints. Int. J. Control Signal Process 17, 195–207 (2003)
4. Yu, S.-Y., Chen, H., Zhang, P., Li, X.-J.: An LMI optimization approach for enlarging the terminal region of NMPC. Acta Automatica Sinca 34, 798–804 (2008)
5. Cannon, M., Deshmukh, V., Kouvaritakis, B.: Nonlinear model predictive control with polytopic invariant sets. Automatica 39, 1487–1494 (2003)
6. Ong, C., Sui, D., Gilbert, E.: Enlarging the terminal region of nonlinear model predictive control using the supporting vector machine method. Automatica 42, 1011–1016 (2006)
7. Yu, S.-Y., Chen, H., Li, X.-J.: Enlarging the terminal region of NMPC based on T-S fuzzy model (submitted, 2007)
8. Chen, H.: Stability and Robustness Considerations in Nonlinear Model Predictive Control. usseldorf: Fortschr.-Ber. VDI Reihe 8 Nr. 674, VDI Verlag (1997)
9. Khalil, H.: Nonlinear Systems (third version). Prentice-Hall, New York (2002)
10. Boyd, S., El Ghaoui, L., Feron, E., Balakishnan, V.: Linear Matrix Inequalities in System and Control Theory. SIAM, Philadelphia (1994)
11. Scherer, C.W., Weiland, S.: Linear Matrix Inequalities in Control, DISC Lecture note, Dutch Institute of Systems and Control (2000)
12. Gao, X.: Control for T-S fuzzy systems based on LMI optimization. Dissertation, Jilin University (2006) (in Chinese)
13. Nesterov, Y., Nemirovsky, A.: Interior point polynomial methods in convex programming. SIAM Publications, Philadelphia (1994)
14. Magni, L., Nicolao, G.D., Scattolini, R.: A stabilizing model-based predictive control algorithm for nonlinear systems. Automatica 37, 1351–1362 (2001)